

Details and prices of publications and copies of past question papers are given in the order form PUBS/27 available on request from The Publications Office, Syndicate Buildings, 1 Hills Road, Cambridge, CB1 2EU.

A series of papers in certain subjects of the General Certificate of Education is published under agreement with the Syndicate by Arnold-ESA, E. J. Arnold & Son Ltd, Parkside Lane, Dewsbury Road, Leeds LS11 5TD, to whom all enquiries should be addressed.

SUMMER 1986

MATHEMATICS 4024/1 (HOME)

SYLLABUS D

PAPER 1

All questions may be attempted.

Answers are to be written on the question paper in the spaces provided, and the question paper is to be handed in at the end of the examination.

If working is needed for any question, it must be shown in the space below that question.

Omission of essential working will result in loss of marks.

NEITHER MATHEMATICAL TABLES NOR SLIDE RULES NOR CALCULATORS MAY BE BROUGHT INTO THE EXAMINATION ROOM.

Questions 1 to 23 carry 3 marks each:

Questions 24 to 28 carry 5 marks each:

Question 29 carries 6 marks.

NEITHER MATHEMATICAL TABLES NOR SLIDE RULES NOR CALCULATORS MAY BE USED IN THIS PAPER.

1 Given that $h = 5$ and $k = -2$, evaluate

(i) $3h + 4k$,

(ii) $2h^2$,

(iii) $h(3k - 10)$.

Answer (i)

(ii)

(iii)

2 Giving each answer as a fraction in its lowest terms, find the exact value of

(i) $2\frac{1}{4} - 1\frac{3}{5}$,

(ii) $\frac{1}{2} \div (\frac{3}{5} + \frac{1}{4})$.

Answer (i)

(ii)

- 3 (a) Express $\frac{2}{5}$ as a decimal, giving your answer correct to 2 decimal places.
 (b) Express $\frac{9}{10}$ as a percentage.
 (c) Express 0.36 as a fraction in its lowest terms.

Answer (a)

(b)

(c)

- 4 (a) Simplify $5p^2 \times 3p^4$.
 (b) Find the number x such that the result of adding x to 40 is the same as subtracting $2x$ from 136.

Answer (a)

(b) $x =$

- 5 (a) A brand of paint contains 0.021 grams of colouring per litre.
 Calculate the mass of colouring in 400 litres of the paint.

- (b) A rectangular flower bed has an area of 7.6 m^2 .

Given that its width is 0.8 m, calculate its length.

Answer (a) g

(b) m

- 6 The triangle ABC is isosceles with $AB = AC$. The bisector of \widehat{ABC} meets AC at D and the bisector of \widehat{BAC} meets BD at X .

Given that $\widehat{ABC} = 80^\circ$, calculate

(i) \widehat{BAC} ,

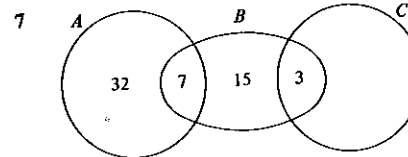
(ii) \widehat{BDC} ,

(iii) \widehat{AXB} .

Answer (i) $\widehat{BAC} =$

(ii) $\widehat{BDC} =$

(iii) $\widehat{AXB} =$



A , B and C are three sets and $S = A \cup B \cup C$.

The numbers of elements in some of the subsets are shown in the Venn diagram and $n(S) = 66$.

Find

(i) $n(A \cup B)$,

(ii) $n(C)$,

(iii) $n(A \cap B')$.

Answer (i) $n(A \cup B) =$

(ii) $n(C) =$

(iii) $n(A \cap B') =$

- 8 List the integer values of x for which

$7x > 65$ and $25 - 2x \geq 1$.

Answer

- 9 (a) Evaluate the matrix product $\begin{pmatrix} 2 & 3 \\ 0 & -4 \end{pmatrix} \begin{pmatrix} -2 \\ 5 \end{pmatrix}$.

- (b) Find the inverse of the matrix $\begin{pmatrix} 4 & -3 \\ 5 & -2 \end{pmatrix}$.

Answer (a)

(b)

- 10 Evaluate

(i) $\sqrt[3]{27}$,

(ii) $\left(\frac{3}{2}\right)^{-2}$,

(iii) $5\frac{1}{2} \times 5\frac{1}{2}$.

Answer (i)

(ii)

(iii)

- 11 (a) Express without brackets in its simplest form $(4 - k)(2 + 7k)$.

(b) Solve the equation

$$\frac{10}{3x} + 1 = \frac{4}{x}$$

Answer (a)

(b) $x =$

- 12 (a) Find the median of the distribution

5.1, 7.9, 3.6, 2.1, 7.9, 4.2, 3.6, 7.8, 3.6.

- (b) The mean of a set of eight numbers is 17.5.

Given that six of the numbers are

12, 14, 15, 19, 24 and 25,

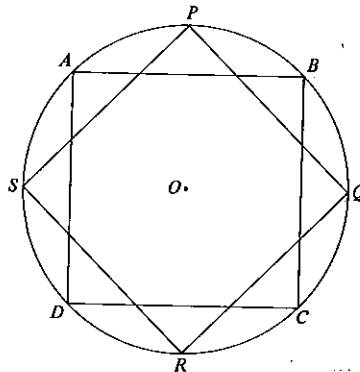
find the mean of the other two numbers.

Answer (a)

(b)

- 13 (a) The angles of a quadrilateral are x° , $3x^\circ$, $5x^\circ$ and $6x^\circ$.
Calculate x .

(b)



$ABCD$ and $PQRS$ are squares inscribed in a circle such that arc $PA =$ arc PB .

The centre of the circle is O .

- (i) State the number of axes of symmetry that the diagram possesses.

- (ii) Given that BC and QR intersect at K , calculate \widehat{POK} .

Answer (a) $x =$

(b) (i)

(ii) $\widehat{POK} =$

- 14 On a map, a length of 4 cm represents an actual distance of 1 km.

Calculate

- (i) the actual distance, in kilometres, represented by 18 cm on the map,
(ii) the scale of the map, in the form $1:n$,
(iii) the area on the map, in square centimetres, which represents an actual area of 3 km^2 .

Answer (i) km

(ii) $1:$

(iii) cm^2

- 15 (a) Using as much of the information below as is necessary, evaluate $\sqrt{34700}$.

$$[\sqrt{3.47} = 1.863, \sqrt{34.7} = 5.891.]$$

- (b) Using as much of the information below as is necessary, evaluate

(i) $\sin 110^\circ$,

(ii) $\cos 130^\circ$.

	20°	40°	50°	70°
sin	0.3420	0.6428	0.7660	0.9397
cos	0.9397	0.7660	0.6428	0.3420

Answer (a)

(b) (i)

(ii)

- 16 (a) Estimate, correct to 2 significant figures, the value of

$$30.02 \times 19.99 - 78.23.$$

- (b) The sides of a rectangle are given as x cm and y cm, where

$$17.5 \leq x \leq 18.5 \text{ and } 11 \leq y \leq 12.$$

Calculate

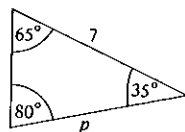
- (i) the smallest possible value of the perimeter of the rectangle,
(ii) the largest possible value of the area of the rectangle.

Answer (a)

(b) (i) cm

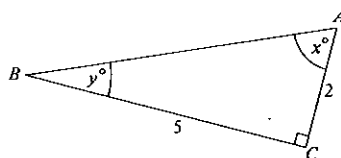
(ii) cm^2

17 (a)



Using the information given in this triangle, write down, but do not solve, an equation which can be used to find p .

(b)



In the given triangle, $AC = 2$ cm, $BC = 5$ cm, $\hat{C} = 90^\circ$, $\hat{A} = x^\circ$ and $\hat{B} = y^\circ$.

Using as much of the information below as is necessary, calculate the value of $x - y$.

$\sin 23.6^\circ = 0.4$ $\cos 66.4^\circ = 0.4$ $\tan 21.8^\circ = 0.4$

Answer (a)

(b) $x - y =$

18 It is given that $y = \frac{k}{x} + 5$, and that $y = 21$ when $x = 3$.

(i) Calculate the value of k .(ii) Hence calculate the value of x when $y = 11$.Answer (i) $k =$ (ii) $x =$

19 (a) A dealer bought a car for £4000 and sold it for £4500.

Calculate his percentage profit.

(b) There are 756 children in a school. This number is 5% more than it was last year.

Calculate the number of children there were in the school last year.

Answer (a)%

(b)

20 (a) $ABCD$ is a parallelogram in which \hat{A} is obtuse and $AB > AD$.

Given that $AB = 9$ cm and that the area of the parallelogram $= 30.6$ cm², calculate the length of the perpendicular from A to DC .

(b) A cylinder has height h and radius r . A sphere has the same radius r .

Given that the volume of the cylinder is twice the volume of the sphere, calculate the numerical value of $\frac{r}{h}$.

(Volume of sphere $= \frac{4}{3}\pi r^3$.)

Answer (a) cm

(b) $\frac{r}{h} =$

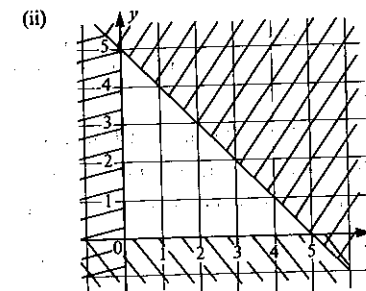
21 The unshaded region in the diagram is defined by three inequalities. Two of these are $x \geq 0$ and $y \geq 0$.

(i) Write down the third inequality.

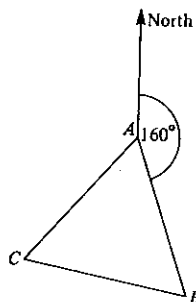
(ii) On the diagram,

(a) draw the line $y = 2x$,(b) indicate clearly, with the letter R , that part of the unshaded region for which $y \geq 2x$.

Answer (i)



22



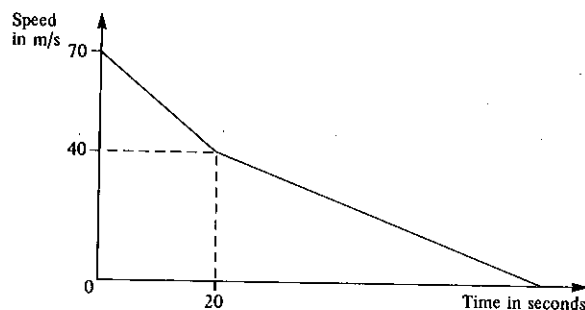
The diagram shows three points A , B and C on level ground at the corners of an equilateral triangle.

Given that the bearing of B from A is 160° , calculate the bearing of

- (i) C from A ,
- (ii) A from B ,
- (iii) B from C .

Answer (i)
 (ii)
 (iii)

23



The diagram is the speed-time graph of a train which is uniformly retarded from 70 m/s to 40 m/s in 20 seconds. The train is then uniformly retarded at 0.4 m/s^2 until it comes to rest. Calculate

- (i) the retardation during the first 20 seconds,
- (ii) the total time that it takes to come to rest,
- (iii) the distance travelled in the first 20 seconds.

Answer (i) m/s^2
 (ii) seconds
 (iii) m

- 24 $A(0^\circ, 35^\circ\text{W})$, $B(0^\circ, 50^\circ\text{E})$, $C(27^\circ\text{S}, 50^\circ\text{E})$ and $D(27^\circ\text{S}, 70^\circ\text{E})$ are four points on the surface of the earth.

- (i) Calculate, in nautical miles, the distance along the equator from A to B .
- (ii) An aircraft flies along the meridian from B to C at 216 knots. Calculate the time taken for the journey.
- (iii) Using as much of the information below as is necessary, calculate, in nautical miles, the distance along the line of latitude from C to D .

$$[\sin 27^\circ = 0.4540, \cos 27^\circ = 0.8910, \tan 27^\circ = 0.5095.]$$

Answer (i) $AB =$
 (ii) time =
 (iii) $CD =$

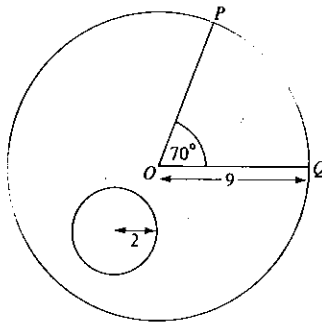
- 25 P is the point (4, 0), Q is the point (10, 4), R is the point (2, 6) and O is the origin.

Find

- (i) PQ^2 ,
- (ii) the gradient of the line QR ,
- (iii) the equation of the line through Q parallel to OR .

Answer (i) $PQ^2 =$
 (ii)
 (iii)

26 (a)



In the diagram, the centre of the larger circle is O , its radius is 9 cm and $\widehat{POQ} = 70^\circ$. The radius of the smaller circle is 2 cm.

A point is selected at random inside the larger circle.

Expressing each answer as a fraction in its lowest terms, calculate the probability that the point lies

- inside the sector POQ ,
- inside the smaller circle.

- (b) In a raffle, 100 tickets are sold of which 50 are red, 30 are blue and the rest are green. After the tickets are thoroughly mixed, one is drawn for the first prize and another is drawn for the second prize.

Expressing each answer as a fraction in its lowest terms, find the probability that

- the first ticket drawn is green,
- the first ticket drawn is red and the second ticket drawn is blue.

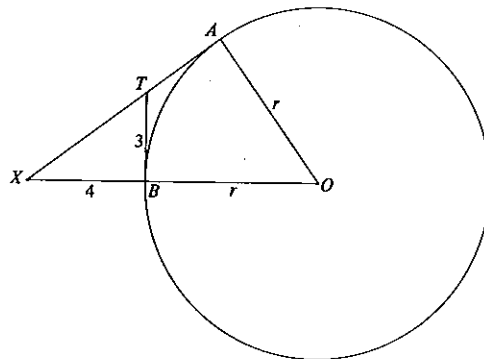
Answer (a) (i)

(ii)

(b) (i)

(ii)

27



TA and TB are tangents to a circle centre O and AT produced meets OB produced at X . $XB = 4$ cm, $TB = 3$ cm and $OA = r$ cm.

- Write down the length of TA .
- Calculate the length of TX .
- Use similar triangles to find the numerical value of $\frac{OA}{AX}$.
- Hence, or otherwise, calculate the value of r .

Answer (i) $TA =$ cm

(ii) $TX =$ cm

(iii) $\frac{OA}{AX} =$

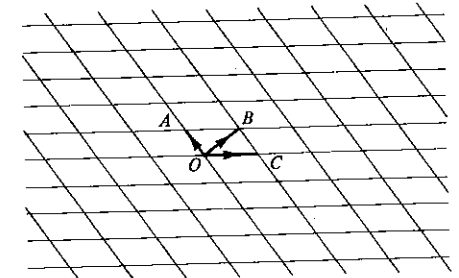
(iv) $r =$

- 28 (a) The diagram shows three vectors \vec{OA} , \vec{OB} and \vec{OC} .

Given that

$\vec{OT} = \vec{OA} + 2\vec{OB} - 3\vec{OC}$,
mark the point T on the diagram, and label it clearly.

Answer (a)



- (b) The column vectors u , v and w are defined by

$$u = \begin{pmatrix} 10 \\ 4 \end{pmatrix}, \quad v = \begin{pmatrix} -2 \\ 5 \end{pmatrix}, \quad w = \begin{pmatrix} p \\ q \end{pmatrix}.$$

- (i) Express $\frac{1}{2}u + 3v$ as a column vector.

- (ii) Given also that

$$u - w = w - v,$$

find the value of p and the value of q .

Answer (b) (i)

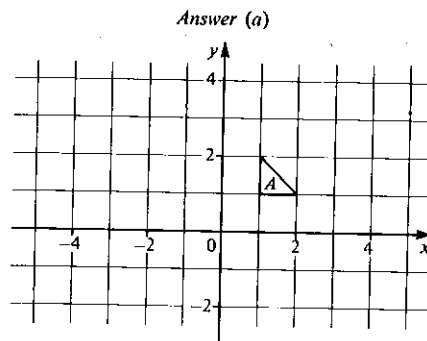
(ii) $p =$

$q =$

- 29 (a) The diagram shows a triangle A with vertices at $(1, 1)$, $(2, 1)$ and $(1, 2)$.

The transformation S is a one-way stretch parallel to the x -axis, leaving the y -axis invariant and with scale factor 2.

On the diagram draw the triangle $S(A)$.



- (b) The transformation T is represented by the matrix $\begin{pmatrix} -3 & p \\ q & r \end{pmatrix}$.

Under T , the point $(1, 8)$ is mapped onto the point $(13, 1)$ and the point $(-3, 4)$ is mapped onto the point $(k, 4)$.

Calculate the values of p , q , r and k .

Answer (b) $p =$

$q =$

$r =$

$k =$

MATHEMATICS 4024/2 (HOME)

SYLLABUS D

PAPER 2

Answer all the questions in Section A and any four questions from Section B.

The intended marks for questions or parts of questions are given in brackets [].

All working must be clearly shown. It should be done on the same sheet as the rest of the answer. Omission of essential working will result in loss of marks.

If the degree of accuracy is not specified in the question and if the answer is not exact, three figure accuracy is required.

Mathematical tables or electronic calculators may be used to evaluate explicit numerical expressions.

Mathematical tables, graph paper and plain paper are provided.

Section A [52 marks]

Answer all the questions in this section.

- 1 2160 people work in a factory and the way in which they travel to work is shown in the table below. No other form of transport is used.

Walk	x
Train	162
Bus	1314
Car	408

- (i) Calculate the value of x . [1]
 (ii) Find the percentage of people who travel to work by train. [3]
 (iii) Of the people who travel to work by bus, the ratio of men to women is 4:5. Calculate the number of women who travel to work by bus. [3]
 (iv) Some of those who travel to work by car share their car with others. Given that the mean number of people per car is exactly 1.7, find the number of people who travel to work as a passenger in a car. [3]

- 2 (a) Given that $a = 3$ and $b = -2$, find the value of

- (i) $(a - b)^3$,
 (ii) $a^3 - b^3$,
 (iii) a^b .

[3]

- (b) Solve the simultaneous equations

$$\begin{aligned} 3x + 2y &= 0, \\ 2x - 3y &= 26. \end{aligned}$$

[3]

- (c) Solve the equations

- (i) $4(2s - 3) - 3(s - 2) = 2$,
 (ii) $\frac{4}{v} = \frac{v}{9}$.

[4]

- 3 (a) Given that $m = 5 \times 10^{-2}$ and that $n = 4 \times 10^3$, calculate, giving each answer in standard form,

- (i) mn ,
 (ii) m^3 ,
 (iii) $\frac{1}{m} + n$.

[3]

- (b) In a survey carried out for a television company, the viewing choices of 100 families, on a particular evening, were recorded.

46 families said they had watched "The Syndicate".
 62 families said they had watched "Casualty Ward".
 13 families said they had not watched either programme.

By drawing a Venn diagram, or otherwise, calculate

- (i) the number of families who had watched both programmes,
 (ii) the number of families who had watched "The Syndicate" only.

[3]

- (c) It is given that $\mathcal{E} = \{x : x \text{ is an integer, } 1 < x \leq 16\}$,
 $A = \{x : x \text{ is a perfect square}\}$,
 $B = \{x : x \text{ is a multiple of 3}\}$,
 $C = \{x : x \text{ is a prime number}\}$.

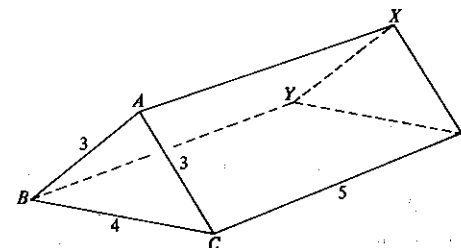
- (i) List the elements of A .

- (ii) Find $n(A \cap B \cap C)$.

- (iii) List the elements of $(A \cup B \cup C)^c$.

[4]

4



The solid triangular prism, shown in the diagram, stands on a horizontal rectangular base $BCZY$. The triangles ABC and XYZ are vertical and the faces $ABYX$ and $ACZX$ are rectangular.

- (i) Given that $AB = AC = 3$ cm and $BC = 4$ cm, show that the height of A above the base is 2.236 cm. [2]

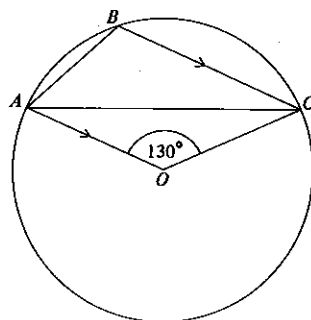
- (ii) Using this height of 2.236 cm and given that $CZ = 5$ cm, calculate, correct to 3 significant figures,

- (a) the volume of the prism, [2]

- (b) the total surface area of the prism. [3]

- (iii) Calculate \widehat{CPZ} , where P is the midpoint of BY . [3]

5 (a)



A , B and C are points on a circle, centre O , such that $\widehat{AOC} = 130^\circ$ and AO is parallel to BC .

Calculate

- (i) \widehat{OAC} ,
- (ii) \widehat{ABC} ,
- (iii) \widehat{BAC} .

[4]

- (b) (i) Construct, in a single diagram,
- (a) the triangle ABC in which $AB = 10$ cm, $BC = 9$ cm and $CA = 8$ cm,
 - (b) the locus of points which are equidistant from B and C ,
 - (c) the locus of points which are equidistant from A and B .
- (ii) Mark clearly, on your diagram, the point X which is equidistant from A , B and C . Measure, and write down, the length XA .
- (iii) The point P , inside $\triangle ABC$, is such that $PA \geq PB \geq PC$. Indicate clearly, by shading, the region in which P must lie.

[8]

Section B [48 marks]

Answer four questions in this section.

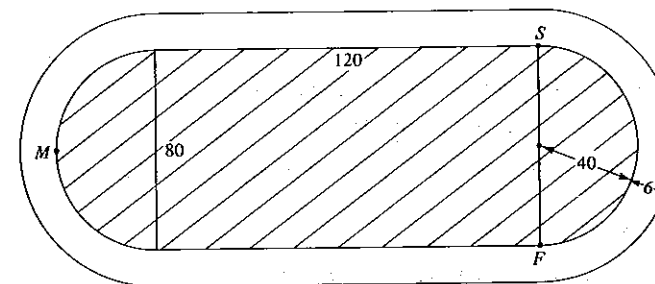
Each question in this section carries 12 marks.

- 6 (a) In a race, the mean time taken by the four runners was 11.13 seconds.

The time taken by the winner was 10.63 seconds and he finished 0.58 seconds ahead of the second runner.

Calculate the time taken by each of the other two runners, given that they finished equal third. [3]

(b)



The diagram shows the plan of a sports ground. The shaded part is made up of a rectangular area 120 m by 80 m and two semi-circular areas of radius 40 m. The shaded area is surrounded by a running track 6 m wide.

- (i) Taking π to be 3.142, calculate

(a) the total shaded area, [3]

(b) the outer perimeter of the running track. [3]

- (ii) S , M and F are three points on the inner perimeter of the track, as shown in the diagram. An athlete ran from S to F via M , along the inner perimeter of the track, at a speed of 5.6 m/s.

Starting at the same time as the athlete, a man walked directly from S to F .

Given that they both arrived at F at the same time, calculate the speed, in m/s, at which the man walked. [3]

- 7 (i) $ABCD$ is a rectangle in which $AB = (4a - 7)$ cm and $BC = (2a - 1)$ cm. Given that $AB - BC = 11$ cm, find the value of a . [2]
- (ii) $PQRS$ is a rectangle in which $PQ = (4p - 7)$ cm and $QR = (2p - 1)$ cm. Given that the perimeter of the rectangle $PQRS$ is 50 cm, write down an equation in p and solve it. Hence find the length of PQ . [3]
- (iii) $WXYZ$ is a rectangle in which $WX = (4x - 7)$ cm and $XY = (2x - 1)$ cm. Given that the area of the rectangle $WXYZ$ is 31 cm^2 , write down an equation in x and show that it reduces to $4x^2 - 9x - 12 = 0$.

Solve this equation, giving your answers correct to two decimal places.

Hence find the length of WX . [7]

- 8 Answer the whole of this question on a sheet of graph paper.

The vertices of the triangle ABC are $A(1, 1)$, $B(1, 4)$ and $C(3, 1)$.

- (i) Using a scale of 1 cm to represent 1 unit on each axis, draw x and y axes for $-8 \leq x \leq 8$ and $-8 \leq y \leq 8$.

Draw and label $\triangle ABC$. [1]

- (ii) An enlargement, centre $(0, 0)$, scale factor 2, maps $\triangle ABC$ onto $\triangle DEF$.

(a) Draw and label $\triangle DEF$.

(b) Write down the matrix which represents this enlargement. [2]

- (iii) A reflection in the x -axis followed by a reflection in the y -axis maps $\triangle ABC$ onto $\triangle LMN$.

(a) Draw and label $\triangle LMN$.

(b) $\triangle ABC$ can be mapped onto $\triangle LMN$ by a single transformation.

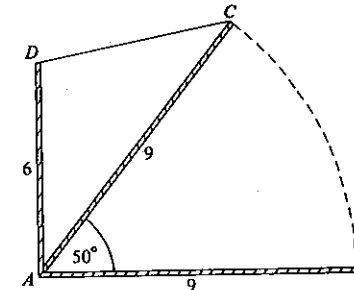
Write down the matrix which represents this transformation. [3]

- (iv) The matrix $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$ represents the transformation which maps $\triangle ABC$ onto $\triangle PQA$.

(a) Draw and label $\triangle PQA$.

(b) Describe fully the transformation. [4]

- (v) Write down the numerical value of $\frac{\text{the area of } \triangle PQA}{\text{the area of } \triangle DEF}$. [2]



The diagram, which is not drawn to scale, represents a crane. The section AD is vertical. The jib, which is hinged at the point A , rotates from the horizontal position AB to the position AC where it is held by a chain DC .

Given that $AD = 6$ m, $AB = AC = 9$ m and $\widehat{BAC} = 50^\circ$, calculate

- (i) the height of the point C above AB , [2]

- (ii) the length of the chain DC , [4]

- (iii) the length of the arc BC , taking π to be 3.142 , [3]

- (iv) the distance BC . [3]

- 10 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation

$$y = \frac{1}{2}(7x - x^2).$$

Some corresponding values are given in the following table.

x	0	1	2	3	$3\frac{1}{2}$	4	5	6	7	8
y	0	3	5	6	a	6	5	3	0	b

- (i) Calculate the value of a and the value of b . [2]

- (ii) Taking 2 cm to represent 1 unit on each axis, draw the graph of

$$y = \frac{1}{2}(7x - x^2)$$

for the range $0 \leq x \leq 8$. [4]

- (iii) By drawing the line $y = x + 1$ on your diagram, find the range of values of x for which

$$\frac{1}{2}(7x - x^2) > x + 1. [2]$$

- (iv) Use your graph to estimate the two values of x which satisfy the equation

$$7x - x^2 = 5. [2]$$

- (v) Estimate, from your graph, the area between the curve $y = \frac{1}{2}(7x - x^2)$, the x -axis and the lines $x = 1$ and $x = 4$. [2]

11 Answer the whole of this question on a sheet of graph paper.

The table below gives the distribution of the weights, in kilograms, of the 60 members of a sports club.

Weight x kilograms	$x < 40$	$40 < x < 50$	$50 < x < 60$	$60 < x < 70$	$70 < x < 80$	$80 < x < 90$	$90 < x < 100$	$100 < x < 110$
Number of members	0	3	6	11	24	10	4	2

(i) Copy and complete the following cumulative frequency table.

Weight in kilograms	40	50	60	70	80	90	100	110
Number of members of this weight or less	0	3	9					

[2]

(ii) Using a horizontal scale of 2 cm to represent a weight of 10 kg and a vertical scale of 2 cm to represent 10 members, draw a cumulative frequency curve for these results over the range $40 \leq x \leq 110$. [2]

(iii) Use your graph to estimate

- (a) the median weight,
(b) the interquartile range.

[3]

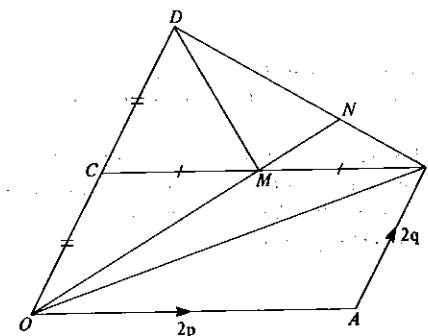
(iv) One member is chosen at random from the 60. Using your graph, estimate the probability that his weight is less than or equal to 54 kg. [1]

(v) Ten of the members are swimmers and the other fifty are athletes.

- (a) Two members are chosen at random from the 60. Find the probability that one will be a swimmer and one an athlete.
(b) One member is chosen at random from the 60. Estimate the probability that he will be a swimmer and will weigh more than 70 kg.

[You may assume that there is no connection between the weight of a member and his sporting activity.] [4]

12



$OACB$ is a parallelogram and M is the midpoint of BC .

OC is produced to D so that $OC = CD$.

OM is produced to meet DB at N .

(i) Given that $\vec{OA} = 2\mathbf{p}$ and $\vec{AB} = 2\mathbf{q}$, express the following vectors in terms of \mathbf{p} and \mathbf{q} , giving each of your answers in its simplest form.

- (a) \vec{OB} ,
(b) \vec{OM} ,
(c) \vec{BD} .

[3]

(ii) Given that $\vec{ON} = h\vec{OM}$ and that $\vec{BN} = k\vec{BD}$, use the fact that $\vec{ON} = \vec{OB} + \vec{BN}$ to write down an equation in terms of h , k , \mathbf{p} and \mathbf{q} .

Hence show that $h = \frac{4}{3}$ and find the value of k . [4]

(iii) Find the numerical value of

- (a) $\frac{OM}{ON}$,
(b) $\frac{\text{the area of } \triangle MDO}{\text{the area of } \triangle MDN}$,
(c) $\frac{\text{the area of } \triangle OBD}{\text{the area of trapezium } OABD}$

[5]